references. This is indeed the case. For a Prandtl number of unity, the table of Ref. 1, with additional temperature ratio values, would be as cited[2] in Table 1. Therefore, as stated in Ref. 1, as the wall is cooled, a consistently larger adverse pressure gradient is required for separation.

References

[1] Morduchow, M., "Review of theoretical investigations on effect of heat transfer on laminar separation," AIAA J. 3, 1377–1385 (1965).

[2] Ball, K. O. W., "Similarity solutions for the compressible separated laminar boundary layer with heat and mass transfer," (to be published).

Comments on "Phugoid Oscillations at Hypersonic Speeds"

Robert J. Woodcock*
Air Force Flight Dynamics Laboratory,
Wright-Patterson Air Force Base, Ohio

Laitone and Chou[1] have presented a good exposition of high-speed longitudinal dynamics, but a few points merit comment (mostly in their notation). In Fig. 1,[1] evidently the artist was given too much liberty. Using the given equations results in the trends of phugoid period shown in this Comment's Fig. 1. It is evident that the simple classical expression, which ignores the density gradient,

$$T = (2)^{1/2}\pi U/g \tag{1}$$

is adequate at approach and landing speeds, and that it remains a fair approximation at all subsonic speeds. The cor-

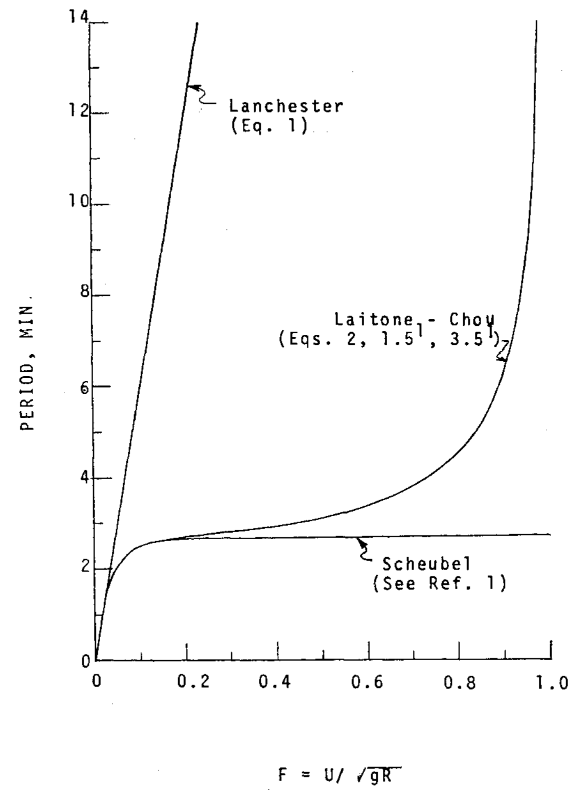

Fig. 1 Phugoid period.

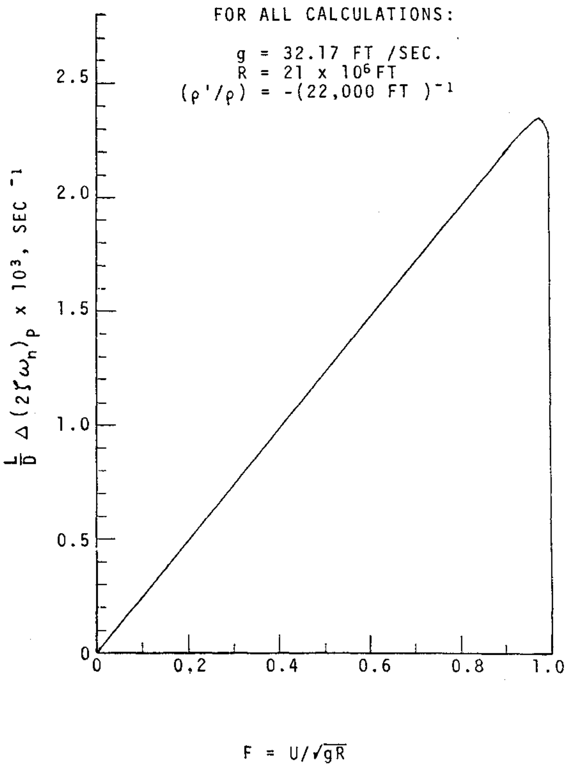

Fig. 2 Phugoid damping factor correction.

rection for earth curvature becomes significant at about the point shown[1] (half the orbital speed) but is smaller than Laitone and Chou indicated at higher speeds. Even so, at all suborbital speeds an excellent approximation is

$$T = \frac{(2)^{1/2}\pi U}{g[1 - (U^2/2g)(1 - F^2)(\rho'/\rho)]^{1/2}} \tag{2}$$

when, as in Ref. 1, derivatives are invariant with Mach number. The drag effect is negligible, except possibly at landing speeds for $L/D < 3$. In any case Laitone and Chou properly apply the drag correction to the period, not to the undamped natural frequency ω_n.

An obvious modification of the damping expression, Eq. (3.8) or (3.9),[1] gives the phugoid-mode damping coefficient

$$2\zeta\omega_n = \left(\frac{\rho US}{m}\right) C_D + 2\left(\frac{C_D}{C_L}\right) \times \left[\frac{-[(\rho'/\rho) + (1/R)]U + 2(g/U)}{-(\rho'/\rho)R + (2/F^2) + [F^2/(1 - F^2)]}\right] \tag{3}$$

making the correction a function of L/D, ρ'/ρ, U, and R. The damping-coefficient correction can then be plotted in the same way as the period (Fig. 2). Of course, in a reentry deceleration or for any nonlevel mean flight path, ζ and ω_n may change markedly during only one cycle of the extremely slow phugoid motion.

Contrary to Laitone and Chou,[1] aerodynamic drag does damp the phugoid oscillation when "the thrust exactly cancels the drag force." A constant engine thrust is equivalent to the gravity "thrust" in gliding flight. The force along the flight path is given by

$$X = T - \tfrac{1}{2}\rho U^2 S C_D - W \sin\gamma \tag{4}$$
$$X_{\text{trim}} = X_0 = 0$$

So, we have the derivative about the trim point

$$\partial X/\partial u = (\partial T/\partial u) - \tfrac{1}{2}\rho_0 U_0^2 S \partial C_D/\partial u - \rho_0 U_0 S C_D = \partial T/\partial u - \tfrac{1}{2}\rho_0 U_0^2 S \partial C_D/\partial u - (2/U_0)(T_0 - W \sin\gamma_0) \tag{5}$$

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* Principal Scientist, Control Criteria Branch, Flight Control Division. Associate Fellow Member AIAA.

In body-fixed axes a similar expression results. Since $\partial X/\partial u$ is the prime contributor, phugoid damping tends to zero not when $T_0 = D_0$, but when $\partial T/\partial u = \partial D/\partial u$.

Finally, the standard definition of a partial derivative holds constant all variables but one. Therefore it appears incorrect to include terms proportional to dr/du in the literal expressions of Table 1[1] for $\partial X/\partial u$ and $\partial Z/\partial u$.

All of these comments leave the general worth of Ref. 1 undiminished. They serve instead to sharpen its focus.

Reference

[1] Laitone, E. V. and Chou, Y. S., "Phugoid oscillations at hypersonic speeds," AIAA J. **3**, 732–735 (1965).

Reply by Authors to R. J. Woodcock

E. V. Laitone* and Y. S. Chou†
University of California at Berkeley, Berkeley, Calif.

The approximation to the phugoid period that is suggested by Woodcock[1] in his Eq. (2) gives the same numerical values as does our[2] Eq. (1.5) within 2% as long as $F < 0.99$, because for the earth's atmosphere ρ'/ρ is nearly 1000 times greater than $1/R$ at altitudes below 400,000 ft. Consequently the additional term in our[2] Eq. (1.5) is necessary only for the orbiting speed ($F = 1$) in order to give the correct earth satellite period. However, for other planetary bodies ρ'/ρ could become of the same order of magnitude as $1/R$, and then the additional term would become very important at hypersonic speeds.

Our[2] Sec. 2 was not based upon a constant engine thrust. As indicated by our Eq. (2.1) we derived Eqs. (1.5) and (2.3) by assuming that a varying thrust continually cancelled the total drag force, so that the condition derived by Woodcock,[1] following his Eq. (5), is automatically satisfied. Our Eq. (1.5) was derived in this manner so as to provide a direct comparison with the previous derivations (based on the same assumption of exactly zero net drag) of Lanchester [Eq. (1.1)] and Scheubel [Eq. (1.2)], and thereby explicitly delineate the new effects indicated by Etkin's[3] numerical calculations. We used Eq. (3.5) for the case of constant engine thrust, and in Sec. 3 we showed that whereas a finite drag force has only a slight effect upon the period, it directly controls all of the damping.

The expressions for X_u and Z_u in our[2] Table 1 must include the term dr/du if the flight trajectory is not parallel to the planet's surface because then r is a given function of u. For the cases considered in Etkin's[3] numerical calculations, the term dr/du can be neglected. However, this may not be the case for any steeply inclined trajectory of an aerodynamic body shape moving at hypersonic speeds through the earth's atmosphere. The term in question can be derived as follows:

$$X_u = -\frac{\partial}{\partial u}\left(\tfrac{1}{2}\rho V^2 S C_D\right) = -\rho U S\left(C_D + \frac{U}{2}\frac{\partial C_D}{\partial u} + \frac{U C_D}{2\rho}\frac{\partial \rho}{\partial u}\right) = -\rho U S C_D\left(1 + \frac{U}{2}\frac{1}{\rho}\frac{d\rho}{dr}\frac{dr}{du}\right)$$

since $\partial C_D/\partial u = 0$ for hypersonic speeds.

Our[2] Fig. 1 was calculated on the basis of the ρ'/ρ values used by Etkin[3] in his numerical calculations, whereas Woodcock's[1] Fig. 1 is based upon a constant value of $\rho'/\rho = -(22 \times 10^3)^{-1}$.

We would like to thank Mr. Woodcock for his comments because they have brought out some interesting details that we had not discussed. We also would like to note two misprints in our Eq. (1.4), which would have been printed as

$$T = (2)^{1/2}\left(\frac{U}{g}\right)\pi\left\{1 + \frac{U^2}{2g}\left(-\frac{\rho'}{\rho}\right) + \left[1 - \frac{U^2}{2g}\left(-\frac{a'}{a}\right)\right]\zeta\right\}^{-1/2}$$

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* Chairman and Professor of Aeronautical Sciences. Associate Fellow Member AIAA.

† Research Assistant in Aeronautical Sciences.

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[1] Woodcock, R. J., "Comments on 'Phugoid oscillations at hypersonic speeds,'" AIAA J. **4**, 762–763 (1966).

[2] Laitone, E. V. and Chou, Y. S., "Phugoid oscillations at hypersonic speeds," AIAA J. **3**, 732–735 (1965).

[3] Etkin, B., "Longitudinal dynamics of a lifting vehicle in orbital flight," J. Aerospace Sci. **28**, 779–788 (1961).

Comment on "Prediction of Adiabatic Wall Temperatures in Film-Cooling Systems"

Geoffrey J. Sturgess*
Loughborough College of Technology, Loughborough, England

Application of Spalding's combined boundary layer and jet-flow model correlation group for film cooling has been made to data from representative aircraft gas-turbine combustion chamber cooling devices, and comparison made between these data and Spalding's equation, which can be taken as applying for clean slot devices. It has been shown that over the appropriate range of values for such application, good correlation is not achieved, and the equation does not describe the data. A significant geometry effect is clearly shown to be present. An alternative correlation group is presented which, although not definitive, gives much improved results, and the resulting equation indicates that treatment of the film as more jet-like might be made for this type of data.

Spalding concludes that an artificial formula,[1] which is an uncomplicated combination of boundary layer and jet-flow model correlations, represents the best simple correlation formula to describe film cooling data over a wide range of velocity ratio. This correlation was tested against, and based on, simple geometry slots, and it was interesting to see how far it could be applied to dirty geometry slots,† which constitute most practical cooling systems. Accordingly, application of this formula has been made to the film cooling of aircraft gas-turbine combustion chambers. For such application, i.e., to a Mach 2.2 cruise supersonic transport typically, values of the group X are unlikely to exceed 10, and the film effectiveness usually must be limited to a minimum of about 0.6–0.7 because of considerations of permitted wall temperature.[2] X‡ was defined as

$$X = 0.91\left(\frac{u_g \cdot x}{u_c \cdot y_c}\right)^{0.8}\left(\frac{u_c \cdot y_c}{\nu}\right)^{-0.2} + 1.41\left(\left|1 - \frac{u_g}{u_c}\right|\frac{x}{y_c}\right)^{0.5} \quad ((1)$$

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* Research Fellow, Department of Aeronautical and Automobile Engineering.

† A dirty slot is defined as one with restricted inlet and/or outlet; it also may protrude into the mainstream. A clean slot is unrestricted and does not usually disturb the mainstream.

‡ Nomenclature as Spalding.